Eur Phys J A **19**, s01, 1–8 (2004)
Digital Object Identifier (DOI) 10.1140/epjad/s2004-03-001-5

EPJ A direct
electronic only

Lattice calculations of hadron properties

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Received: 13 Aug 2003 / Accepted: 14 Nov 2003 /
Published Online: 6 Feb 2004 –

Abstract. Recent lattice studies of hadron properties, in particular of exotic states and charmonia are reviewed. Sea quark and quark mass effects are discussed as well as decays and mixing.

PACS. 12.39.Mk Glueball and nonstandard multi-quark/gluon states – 14.40.Cs Mesons with $S = C = 0$ – 14.40.Gx Mesons with $S = C = B = 0$ – 12.38.Gc Lattice QCD calculations

1 Introduction

While the simplicity and elegance of QCD is very appealing theoretically, the phenomenological observations of spontaneous chiral symmetry breaking and even more so of the confinement of colour charges turned it into a major calculational nightmare: it took almost twenty years after the discovery of asymptotic freedom to convincingly demonstrate that the QCD Lagrangian indeed implies these highly non-trivial collective phenomena. This was done by numerical simulations; an analytic proof is still lacking.

Fortunately, due to the property of asymptotic freedom, many short distance/high energy QCD problems can be approached by means of perturbation theory. This need not be so since the very basis of the perturbative expansion is shaky: confinement implies that quark and gluon fields never appear as asymptotic states. Fortunately, the success of jet phenomenology suggests that QCD is reasonably benign in the high energy region. This is very different in the low energy regime of *strong QCD*. Among the few analytical tools that exist are the strong coupling and the $1/N$ expansions, effective field theories (EFTs) and various QCD inspired or phenomenological models.

QCD can in principle be solved rigorously by means of lattice simulations on a computer. In practise however computational resources are finite. This leads to pion masses of typically more than 400 MeV or to simulations within the quenched approximation, where sea quark effects are neglected. While these systematic uncertainties reduce with faster computers, improved numerical algorithms and theoretical ingenuity, it will always remain desirable to combine lattice simulations with EFT methods or, where necessary, with QCD motivated models as only relatively simple questions can directly be addressed on the lattice. In many cases problems of phenomenological interest factorise naturally into a high energy electro-weak part and a low energy QCD part which can then be evaluated on the lattice, for instance electromagnetic form factors and weak decay matrix elements. In many cases QCD problems can also be factorised within the framework of EFTs into low and high energy parts. Lattice simulations turn out to be invaluable to gain insight into the dynamics of QCD as many parameters such as the number of active quark flavours, quark masses, number of colours, temperature etc. are not limited to their phenomenological values but can be varied.

Several good reviews of lattice calculations exist in the literature and I refer to them for details on theoretical aspects, calculational methods and wider phenomenological implications [1]. The topics covered in Chris Michael's review [2] "Exotics" have a non-vanishing overlap with this article, albeit written from a (slightly) different perspective. In my talk I covered the baryon spectrum and structure as well. Due to the page limit (and lack of new content) I refer to my recent review [3]. Subjectively selected highlights since then include two studies of generalised parton distributions [4,5] and a determination of the pion form factor [6]. Some progress has been made in the calculation of electromagnetic N to Δ transition form factors [7], a new study of the mass spectrum of excited nucleons has appeared [8] and a first step towards consolidation of previous results was performed [9]. I will not mention the Θ^+, because there are no lattice results. Instead I will concentrate on the spectroscopy of charmonia and exotics.

2 Glueballs and friends

It was realised as early as 1974 [10] that QCD offered the possibility of bound states composed only of energy. These closed-string-/glue-states/boxcitons became subsequently known as glueballs [11]. It is hard to imagine anything that demonstrates confinement more than the discovery of quarkless massive bound states. One can also fantasize that non-perturbative physics with particles similar to glueballs might play a rôle in future theories *beyond the Standard Model*. After all QCD is the only part of the Standard Model with a chance of a mathematically rigorous definition. So glueballs, the simplest possible colour-neutral states, are a challenge that certainly has to be addressed.

The situation is complicated theoretically as well as experimentally by the possibility of mixing with standard quark model states. Even worse QCD does not know what is *exotic* and what is not. The very meaning of this term is *"not quark model"* and it is not at all *a priori* clear what exactly we mean by *quark model*. For instance we can ask ourselves how closely the quark model describes something as profane as the QCD proton. While introducing the concept of constituent quarks eliminates the puzzle that more than 98 % of its rest mass is generated by spontaneous chiral symmetry breaking, i.e. by the glue, it is impossible to attribute the proton's spin and momentum exclusively to quark degrees of freedom. Gluodynamics plays an even greater rôle in properties of the η' and the π. So if the gluons already leave big footprints on *quark model states* how can we distinguish these from *glue states*? One possibility that immediately springs to mind would be to look for *spin-exotic* states: when coupling spin and angular momentum within mesons, only certain combinations of J^{PC} are allowed while for instance $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ are forbidden. Again the situation is complicated by the possibility of $q\bar{q}q\bar{q}$ molecular states to couple to these same quantum numbers.

One toy model is the pure Yang Mills theory of gluodynamics. In this fictitious world no quark fields exist but many central features of QCD including confinement and asymptotic freedom are still reproduced. This well defined and self consistent theory yields a rich spectrum of glueballs, all of which are absolutely stable, with the exception of very heavy ones that decay into lighter glueballs.

One can go one step further towards QCD by including quark fields that propagate in the gluodynamic background but whose feedback onto the vacuum is neglected, the so-called quenched approximation. Quenched QCD (qQCD) is no quantum field theory since unitarity is violated, however, even in this approximation chiral symmetry is spontaneously broken. Neglecting the feedback of slowly moving heavy quark sources onto their environment is a very natural thing to do. Quenching is also justifiable in the limit of the number of colours $N \to \infty$, however, it is not always clear whether $1/3 \ll 1$. Simulations of pure $SU(N_c)$ gauge theories for [12,13] $N = 2, \ldots, 6$ seem to indicate this. As it should be ratios of light hadron masses from lattice simulations of qQCD have been found to be inconsistent with the observed spectrum [14] however the differences are typically smaller than 10 % suggesting that the quenched approximation has some predictive power if cautiously consumed. The consequences of violating unitarity at light quark mass can become dramatic in some channels and in particular in the scalar sector [15]: roughly speaking as the axial anomaly does not exist in qQCD the η' will be a surplus light Goldstone boson. The impact of this can be investigated in quenched chiral perturbation theory where diagrams that include transitions from scalar to two πs yield unwanted contributions that explode as the πs, including the would-be η', become light.

One justification for quenching is that the computational effort is easily reduced by a factor of 10^3. So it makes complete sense to learn and to understand statistics and systematics from a quenched study, prior to doing the real thing. Moreover, most models used in hadron physics neglect quark pair creation and annihilation, so the model builders can still learn from qQCD. Also, "un-quenching" a model to compare it with lattice results is easier than unquenching the lattice simulation. Last but not least, life is easier in quenched as lattice studies of strong decays and excited states are notoriously complicated: glueballs will not mix with quark model mesons, spin exotic meson-gluon hybrids are distinct from mesons, are stable and do not mix with four-quark molecules either.

2.1 Heavy glueballs and charmonia

In Fig. 1 we compile the glueball spectrum (filled boxes) of gluodynamics [13,16]. The scale $r_0^{-1} \approx 394$ is set from potential models. Since this is not the real world a systematic scale error of about 10 % should be assumed, for stable states that do not mix! While these newer results agree with the spectrum of [17], during the past decade the statistical errors have been reduced by factors varying between 1.5 and 2, depending on the channel. It has also been established that the 3^{++} glueball is lighter than the 1^{++}. This had only been suspected in the earlier reference, where the J assignment of this state remained ambiguous.

The lightest glueball is a scalar, followed by a tensor and a pseudoscalar. Possibly the lightest state with exotic quantum numbers is a 2^{+-} glueball around 4 GeV. Qualitative features can be understood in a bag model while the flux tube model seems inadequate [18]. Without numerical simulation it would have been completely impossible to come close even to a semi-quantitative understanding of these truly unconventional bound states.

We include experimental charmonium levels (lines) for comparison. The ${}^{2S+1}L_J$ notation refers to these states. We have only included confirmed resonances, with masses taken from the Particle Data Book. For the η_c' we took the Belle result from double charmonium production [19], 3630(8) MeV. One often thinks of the charm quark as heavy in the sense that its mass is much bigger than mesonic and baryonic QCD binding energies. However, the spectrum of states entirely made out of glue with no quarks at all covers a similar energy range! We also display recent quenched lattice results from two groups [20,21,22] (CP-PACS and Columbia). These have been obtained using relativistic charm quarks on anisotropic lattices with the ratio of spatial over temporal lattice spacings $\xi \approx 2$ (Columbia) and $\xi \approx 3$ (CP-PACS). A recent simulation of $J \leq 1$ S and P wave charmonia on isotropic ($\xi = 1$) lattices by QCD-TARO [23] (who also study charmonium

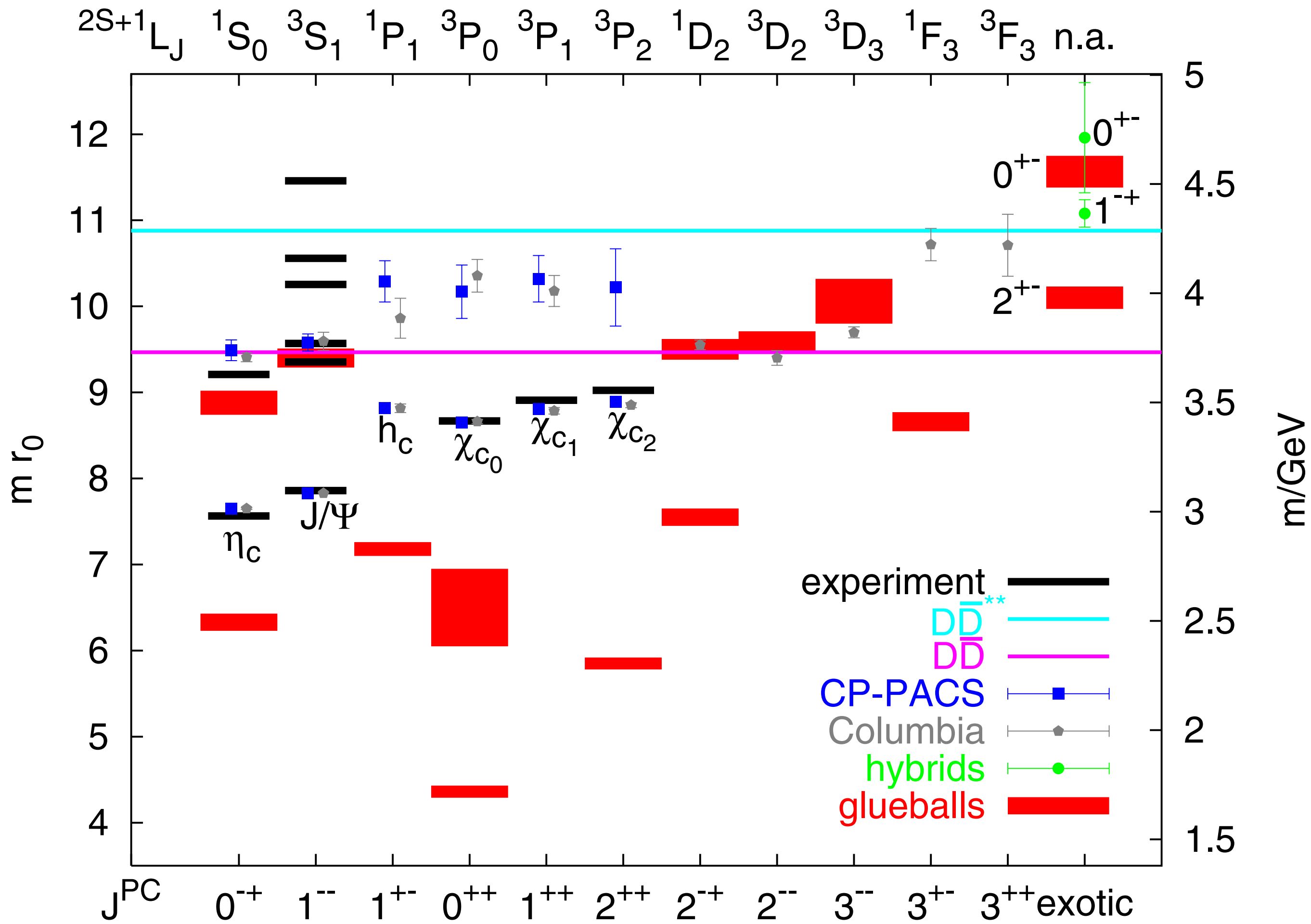

Fig. 1. The quenched charmonium spectrum (CP-PACS [20], Columbia [21,22]), glueballs [16,13] and spin-exotic $c\bar{c}$-glue hybrids [22], overlayed with the experimental spectrum

wave functions) confirms these findings. Note that in all these simulations the effect of diagrams with disconnected quark lines has been neglected. One might expect OZI violating contributions from these, in particular for states that lie close to glueballs with the same quantum numbers.

The charm quark mass has been adjusted such that the spin averaged $1S$ state reproduces experiment. This means that in this case the quenching errors of up to 10 % apply to spin-averaged level splittings with respect to this ground state, rather than with respect to zero energy. This systematic scale uncertainty caused by omitting sea quarks renders it irrelevant whether we use a "constructed" scale like r_0 or an experimental mass like the $1P-1S$ gap as an input (which would increase the splittings by about 6 %). For the fine structure, potential models combined with lattice results [24] tell us that we should expect to undershoot the real world number by up to 40 %. For states above threshold or where mixing effects and strong decays play a big rôle radical changes might occur while for the $2P$ excitations finite volume effects could be an issue.

There are different ways of obtaining a given J^{PC}: for instance $J^{PC}=1^{--}$ can be an S or a D wave. The correlation function associated with the respective D wave operator decays very fast in Euclidean time, into the same ground state as the respective S wave: it appears that the charm quark is too light to turn L into an (approximately) good quantum number. A similar behaviour has been observed for the 2^{++} and 1^{++} states (either P or F waves).

In addition to the standard charmonia and glueball states the figure contains the lightest two spin exotic $c\bar{c}$-gluon hybrids [22]. At least two other studies of hybrid charmonia with relativistic charm quarks exist to-date [25, 26]. Given the fact that not even L is a good quantum number it is not clear how one would distinguish non spin-exotic hybrids from conventional radial excitations. The lightest exotic hybrid turns out to be a vector, 1^{-+}. This is followed by 0^{+-} and 2^{+-} hybrids.

Once sea quarks are switched on, most of the states calculated on the lattice will decay strongly and mixing will occur too. In this context it is interesting to see that the spin-exotic 0^{+-} glueball and $c\bar{c}g$ hybrid have similar masses. We have included two experimental thresholds into the figure: 1^{--} charmonia can and will decay into a $D\overline{D}$ meson pair. Nonetheless resonances that exceed this threshold by almost 1 GeV have turned out to be experimentally detectable. The exotic 1^{-+} hybrid cannot decay into an identical boson-antiboson pair since $PC = -$. The next possibility, a $\overline{D}$ and a vector D^*, is suppressed both in flux tube models [27] where the quark-antiquark pair that breaks the flux tube is produced in its centre, as well as in the heavy quark limit in which the angular momentum of the heavy quarks is fixed. The next possibility, which follows this $S + P$ selection rule, would be a decay into a $\overline{D}$ and a P wave isovector $D^{**}[= D_1(2420)]$. Interestingly the mass of the 1^{-+} is compatible with this threshold such that mixing with (would be) molecular states is an issue. Should the state be below this threshold a resonance with a width smaller than 100 MeV might be possible, with dominant decay into a χ_c under emission of a light scalar meson [28].

At present we do not know whether the overlap with glueball states in the quenched approximation has any implications on the phenomenology of charmonia. Naïvely the phase space for decay of heavy glueballs appears huge with no quarks to preserve and more than 3 GeV of energy to disperse. However, before drawing definite conclusions dynamical issues related to the glueball wave functions need to be investigated. In particular mixing is a strong possibility: for instance the $\psi(3770)$, $\psi(2S)$, $D\bar{D}$ threshold and the vector glueball all lie within the same 100 MeV mass window, allowing for a potentially interesting phenomenology, in particular since there is little evidence from lattice simulations to support distinguishable $2S$ and $1D$ levels in this region. Recently, the interesting question of a (non-exotic) hybrid component in the $1S$ states has been addressed in a very nice study [29], in the framework of NRQCD. The conclusion is that this contribution is weak. The question becomes more exciting but also tremendously less approachable if the $2S$ charmonium state and glueball channels were considered in addition.

With data from Belle and Babar emerging, CLEO-c being online and the possibility of BES III starting to take data in 2006 the charmonium region is also exciting experimentally. Unfortunately, there is no way of producing say spin-exotic 1^{-+} states at any detectable rate in a decay starting from a vector resonance. For this we might have to wait for the proton-antiproton PANDA experiment at GSI. However, in the meantime there will definitely be progress in measuring spectrum and decay rates. Possibly the glue-richness of this mass region will leave its imprint, for instance by enhancing OZI suppressed processes.

2.2 Light scalars: Today

A lot of experimental attention has been devoted to the spectroscopy of light scalar mesons. The reasons are threefold: the scalar sector is intimately linked to chiral symmetry breaking which results in both light πs but at the same time also in heavy σs. Furthermore, the lightest glueball is predicted to be a scalar and, as an added extra, the lightest four-quark candidate state has scalar quantum numbers as well: pseudoscalar-pseudoscalar bound states can be lighter then quark model scalars since chiral symmetry breaking makes the latter "artificially" heavy and the former "artificially" light. It is also not completely accidental that gluons play a dominant rôle in both the pseudoscalar and the scalar sectors, in the former due to the axial anomaly and in the latter due to glueballs, both of which result in large OZI violations and splittings between $I = 0$ and $I \neq 0$ mesons. Unfortunately, it is exactly this rich phenomenology which complicates the interpretation of experimental results as well as theoretical calculations.

The lightest scalar "particle" is the σ [or $f_0(600)$]. It seems by now clear that this pole shifts the phase of the $\pi\pi$ S wave, both from $\pi\pi$ scattering experiments as well as from proton-antiproton collisions with a 3π final state. With an imaginary part of the T matrix pole of 450(150) MeV, which almost exceedes its real part [800(400) MeV], few people are left who would call this sort of "resonance" a "particle". However historically, before the advent of lattice simulations, the σ was among the first glueball candidates. Next there is the $f_0(980)$ which in contrast has a width not much larger than 50 MeV, and is accompanied by the $I = 1$ $a_0(980)$ triplet. This "narrowness" is due to the fact that these states cannot decay into $K\overline{K}$, being 10–15 MeV lighter than this threshold. This proximity also turns them into natural candidates for either $s\bar{l}l\bar{s}$ or $K\overline{K}$ bound states. For simplicity We will not distinguish between four-quark and meson-meson bound states but refer to both as "molecules".

Lattice results [17,30,16,13] suggest that the lightest glueball should be a scalar with mass somewhere between 1.4 and 1.8 GeV. While all raw lattice data agree within statistical errors of some 40 MeV, the large uncertainty quoted above is due to the scale uncertainty from using the quenched approximation. These results lie on top of three experimental scalar $I = 0$ resonances, the broad $f_0(1370)$, the extremely well studied $f_0(1500)$ and the $f_0(1710)$. The standard picture is that these states are mixtures between a glueball and two $I = 0$ nonet mesons, one with dominantly $u\bar{u} + d\bar{d}$ and the other with $s\bar{s}$ quark content. The seven remaining $I \neq 0$ nonet members are most likely the four $K_0^*(1430)$ as well as three $a_0(1450)$ states. Whether there is an excess of experimental states over quark model states or not critically depends on how resonances are organised into nonets. It has to be said that the $a_0(1450)$ is quite broad and that the experimental evidence for these states is not rock hard. On the other hand, with a splitting of 450 MeV, it is extremely hard to reconcile the K_0^* and the $a_0(980)$ into the same nonet, in particular the near degeneracy of the a_0 with the $f_0(980)$ suggests very tiny OZI violating effects. [This in turn also means that there is little chance for a significant glueball component in the $f_0(980)$.] The remaining puzzle is the absence of $S \neq 0$ partners of the $a_0/f_0(980)$. Most likely these states have a large molecular component and are part of an inverted nonet [31], together with the $f_0(980)$, a would-be-$\pi\pi$ σ state and would-be-πK κs, which just happen to be extremely fragile, due to the light π involved.

2.3 Light scalars: Future

To support the above picture an unambiguous experimental identification of the $a_0(1450)$ and a theoretical clarification of the molecular nature of the $a_0/f_0(980)$ states rank high on the wish list. Different experimental and theoretical inputs have led to various mixing models [32, 33,34,35]. Amsler and Close [32] suggest that the light quark state mainly goes into the $f_0(1370)$, with a subleading component mixing into the $f_0(1710)$ while the $s\bar{s}$ and the glueball are mainly distributed between the $f_0(1500)$ and $f_0(1710)$. This view has been superceeded by Close and Kirk's more recent analysis of decays and production rates [35] which suggests the $f_0(1500)$ to be dominantly glueball in character with light and strange quark components mixing destructively and additively into the $f_0(1370)$ and $f_0(1710)$, respectively. In contrast [33,34] predict the $f_0(1710)$ to be the (dominant) glueball-state with the two lighter resonances being composed primarily of the quark model mesons.

All these predictions crucially depend on the input parameters used, in particular on the ordering of "unmixed" states and on assumptions on the mixing matrix. One might expect the unmixed $d\bar{d}+u\bar{u}$ state to have a mass close to that of the $a_0(1450)$ but unfortunately the latter resonance is very broad experimentally and not overly well established. While in QCD with light sea quarks all states will automatically be "mixed", the quenched approximation is ideal to address the question of ordering of "unmixed" states and even for estimating off-diagonal mass-matrix elements. The main problem however is that in this case correlation functions will loose positivity at small quark masses as the π and the η' are degenerate [15]. In practise this means that at best $s\bar{s}$ scalars can be approached using traditional methods.

In [34] significant finite size effects and a strong lattice spacing dependence are observed. After extrapolations the quenched $s\bar{s}$ scalar appears to be about 300 MeV lighter than the pure glueball. RBC [36] quote an even lower mass of 1.04(7) GeV for the light quark a_0 and 0.9(1) GeV for the $I=0$ f_0/σ. The leading chiral correction from the $\pi\eta'$ loop was included into this study. One would however hope that systematic uncertainties will be identified and investigated in the future. Should this result be confirmed, then the $a_0(980)$ could be explained as a quark model state but what is the $f_0(980)$ and why are the K_0^*s, that are experimentally well established, so heavy? In a very sophisticated study Bardeen *et al.* [37,15] determine the relevant low energy parameters of the quenched chiral Lagrangian and remove the leading order quenched chiral power and log. After testing the approach in the pseudoscalar sector they are able to determine an a_0 mass of 1.33(5) GeV, expressed in units of the ρ mass, which is by about 200 ± 100 MeV lighter than the glueball, extrapolated to the continuum limit and converted into the same units. The unmixed $s\bar{s}$ state might then be expected to be fairly equal in mass to the glueball. The main difficulty is that with only two lattice spacings at hand the extrapolation to the continuum limit is still a bit shaky.

The situation has also been studied by UKQCD [38] in QCD with sea quarks at lattice spacings $a \approx 0.1$ fm and $a \approx 0.13$ fm. In this case the a_0 is significantly lighter but the same holds true for the flavour singlet meson/glueball (see Fig. 2 below).

In addition to the studies of quark model scalars some results exist on four quark molecules, but with degenerate masses, with the aim of addressing the possibility of a $\pi\pi$ bound state. Results of quenched studies [39] are inconclusive due to the requirement of large volumes to avoid squeezing the πs on top of each other when they intend to be elsewhere: without a careful finite size study a residual interaction energy can hardly be disentangled from the $\pi\pi$ scattering phase [40]. Diagrams with disconnected quark lines have also been neglected (with one exception [41]). In the absence of definite results in the quenched approximation, the SCALAR collaboration has started the very ambitious program of determining the mass of an $I=0$ $\pi\pi$ state incorporating sea quarks [42]. In view of the above it is highly attractive to study simplified cases, with heavier quarks and $I=1$, like $K\overline{K}$ or DK (see also Sect. 2.4 below) molecules, both with at least equally important phenomenological implications.

Few exploratory studies of strong decays exist. Since we are working in Euclidean time there is no concept of asymptotic states. Neither can we calculate the imaginary part of a forward amplitude but there are bug-fixes: the method of choice is the computationally very challenging exploitation of finite size effects introduced by Lüscher [40]. The poor man's approximation has been developed by Gottlieb *et al.* [43] and Michael [44]. In the latter case an on-shell transition matrix element is evaluated by adjusting the rest mass of the decaying particle to the energy of the outgoing state. In QCD with sea quarks this method can only be approximate as the operator used to create the initial state will in general also have a non-vanishing overlap with the final state. The transition matrix element is related to a coupling which in turn, assuming momentum independence, can be normalised to phase space, predicting the partial decay width. The on-shell condition implies that for a given mass of the final state the relative momentum has to be adjusted to guarantee energy conservation. As momentum is discretized on a lattice this imposes some constraints. The initial state can be boosted such that at a fixed mass several relative momenta can be (approximately) realised, on sufficiently large volumes, and the momentum dependence of the coupling checked. Chiral perturbation theory can also help to connect results obtained at different quark mass, once these are sufficiently light.

Promising results using this approach have been obtained recently for the $\rho \to \pi\pi$ decay in $n_f=2$ QCD by McNeile and Michael [45]. Decay couplings of a glueball to two pseudoscalars have so far only been computed by GF11 [30] some 9 years ago: a mass dependence has been observed with a stronger coupling to heavier mesons, which then has to be folded with phase space.

Elements of the glueball-scalar mass matrix have been calculated by two groups, again GF11 in qQCD [34] and UKQCD for $n_f=2$ [46], for quark masses around the strange quark. Both collaborations obtain mixing energies of about 300 MeV and 500 MeV on relatively coarse lattices, respectively. Starting from two degenerate unmixed states this would imply level splittings between the mixed states of 0.5 – 1 GeV. An extrapolation to the continuum limit by GF11 who also simulated three finer lattice spacings resulted in 61(45) MeV, a very reasonable value but with a 100 % uncertainty.

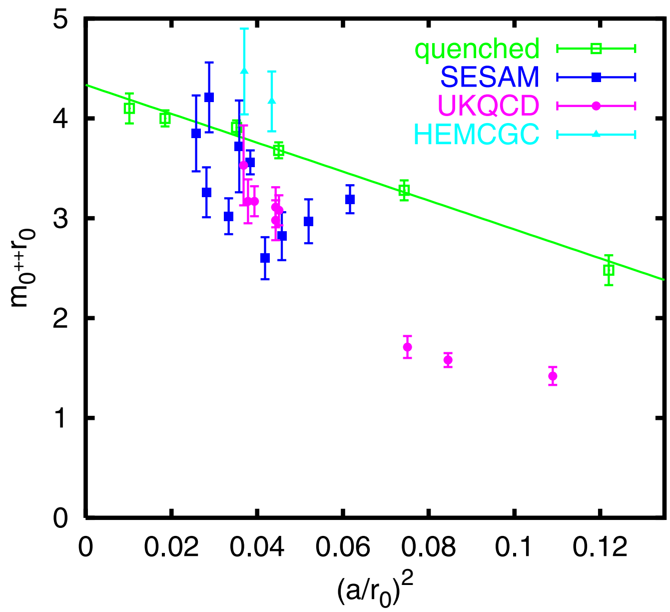

Fig. 2. The scalar "glueball": qQCD vs. $n_f = 2$

We are still in the position that the combined "world data" on the scalar $n_f = 2$ "glueball", plotted as a function of the squared lattice spacing a^2, fits into Fig. 2. The quenched case [13,17] is included for reference. The un-quenched results have been obtained by use of three different lattice discretizations of the Dirac action: staggered (HEMCGC [47]), Wilson (SESAM [48]) and clover (UKQCD [46,38]). The quarks are all not much lighter than the strange quark, the scalar meson is still stable and the wave function turns out to be very close to that of the quenched glueball [48,46]. Most $n_f = 2$ points clearly lie below the quenched line, however, there is certainly a slope in the results, such that the mass in the physical $a = 0$ limit does not contradict the quenched result. Within the SESAM data set there is an apparent discontinuity because different points have been obtained at different quark masses; the "glueball" becomes lighter as the quark mass is reduced. Clearly additional studies at lighter quark masses and different lattice spacings are required.

2.4 The $D^+_{sJ}(2317)$

Recently a narrow $D^+_{sJ}(2317)$ state has been detected by BaBar [49], dominantly decaying into $D_s\pi$. This finding was confirmed by CLEO [50] and by Belle [51]. The latter two collaborations also reported a narrow resonance around 2537 MeV, decaying into $D^*_s + \pi$. Both states lie by about 40 MeV below the respective DK and D^*K thresholds. In contrast potential model calculations suggest [52] much heavier masses for the missing D_{s0} and D'_{s1} P wave states, rendering these into broad resonances. These expectations have very recently received experimental support from Belle's observation of the missing D^*_0 and D'_1 states at 2308(17)(15)(28) MeV and at 2427 (26)(20)(15) MeV, respectively [53]. These states indeed strongly decay into $D\pi$ and $D^*\pi$, respectively, with widths of order 300 MeV.

If the new D_{sJ} mesons were P wave $c\bar{s}$ mesons, why are they so light but the corresponding $c\bar{d}$ mesons are not? Naturally this question invites speculation that the new scalar state might be of a DK (or 4-quark) molecular nature, somewhat resembling the $f_0/a_0(980)$ system [54]. Of course such ideas eventually have to be substantiated by a QCD calculation and indeed lattice results exist: simulations in the static limit within the quenched approximation [55] and with sea quarks [56], simulations including NRQCD/HQET $1/m$ corrections [57] and simulations with a relativistic charm quark [58,59]. The interpretation of these results is controversial: I [56] observed that both effects, including sea quarks and including relativistic corrections to the static limit, increase the mass of a quark model $c\bar{s}$ P wave scalar state, pushing it above the DK threshold and into agreement with potential model predictions. I then concluded that lattice results are incompatible with a pure quark model nature of the new D_{sJ} states. On the other hand, while confirming an increase in the predicted mass when incorporating sea quarks, the authors of [59] conclude that their results are consistent with the states observed by Babar and CLEO, however, within errors no disagreement is seen with the potential model predictions either. Interestingly, while $1/m$ corrections to the static limit are substantial for charm quarks and increase the $0^+ - 0^-$ splitting by about 25 % [56], the $1^+ - 1^-$ splitting is found to agree with the $0^+ - 0^-$ splitting within statistical and systematic errors of about 15 %, in quenched as well as with sea quarks [59], as suggested by chiral symmetry in the heavy quark limit [60].

The same chiral symmetry argument would also apply to molecular states, such that the mere discovery of similar B_{sJ} mesons would not help in discriminating between meson and molecule. Dynamical issues need to be addressed and decays investigated. In particular electromagnetic decays, where the theoretical understanding is much better than for strong decays, would reveal information about the internal structure of these states. The narrowness of these resonances suggests that this need not be a completely hopeless enterprise. If the lightest D_{sJ} state was dominantly a molecule, then a quark model $J = 0$ resonance should exist in addition. The recent Belle [53] evidence of the $D^*_0(2308)$ suggests that another broad resonance, centred around 2.4 to 2.5 GeV with dominant decay into DK, might be detectable.

Better lattice studies are needed to clarify these important issues, and also with respect to the possibility of similar states in the B meson system. Of course at physical light and strange quark masses lattice QCD should reproduce the experimental spectrum. So to gain insight into the nature of these states a high precision quenched benchmark study is required: in this approximation molecules and mesons are clearly distinct. In addition, the light sea quark mass dependence of both, a possible $c\bar{l}l\bar{s}$ molecule and a $c\bar{s}$ meson has to be traced down into the region where mixing can set in. Fortunately, the DK system is more user friendly than its $K\overline{K}$ counterpart: heavier

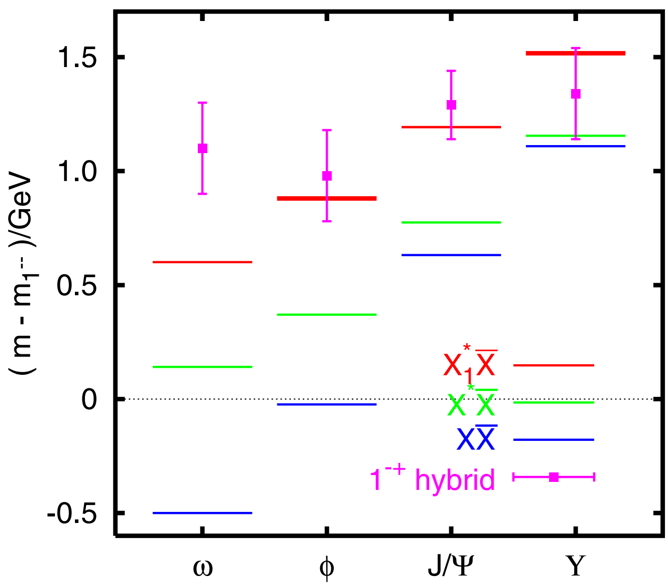

Fig. 3. Splitting of hybrid meson masses with respect to the respective triplet S wave ground states for the light, strange, charm and bottom cases. X denotes π, K, D and B mesons. X^* denotes the ρ, K^*, D^* and B^* vector mesons while X_1^* stands for the (likely) 1^+ states $b_1(1235)$, $K_1(1270)$, $D_1(2400)$ and $B_J^*(5732)$, respectively

particles can be accommodated in smaller volumes and the binding energy that is suggested by phenomenology is comfortably large with about 40 MeV, rather than a mere 10–15 MeV. It is also conceivable to calculate matrix elements that are related to electromagnetic decay rates.

2.5 More hybrids

The spectrum of $c\bar{c}$ glue hybrids has already been discussed in Sect. 2.1 above. Light hybrids have been studied to some extent as well, with [61,62] and without [63, 25,26] sea quarks. All these results yield the same ordering as in the charmonium case (and in fact also the bottomonium case) with 1^{-+} being the lightest exotic, followed by 0^{+-} and 2^{+-}. The 1^{-+} is consistent with a mass of 1.9(2) MeV, with the corresponding strange quark exotic about 200 MeV heavier, in quenched as well as in un-quenched simulations. This is heavier than the $\pi_1(1600)$ candidate. Mixing with molecular states is a possible explanation and the feasibility of studying this has been demonstrated by MILC [26]. However, at present the quarks are still so heavy that for instance the combined mass of a b_1 and a π is around 1.9 GeV too. Clearly the lattice calculations are incomplete. Molecules have to be included and the quark mass dependence of the mixing matrix has to be studied carefully.

Heavy hybrids can be studied using NRQCD [64, 65,66,67,68,29,18] or the Born-Oppenheimer approximation [69,64,67,28,18]. In the charmonium case simulations with relativistic quarks on anisotropic lattices have been pursued by several groups [22,23,25] as well as with isotropic lattices [26,62]. Even a simulation of bottomonium with relativistic quarks on an anisotropic lattice exists [70]. With the statistical errors of present simulations differences between quenched and un-quenched data cannot clearly be resolved. In Fig. 3 We summarise the present estimates of the splitting of the 1^{-+} hybrid with respect to the respective vector meson state: the flavour dependence is tiny. In addition three decay thresholds are displayed. As detailed in Sect. 2.1 a strong decay into two pseudoscalars is forbidden while the decay into vector and pseudoscalar is suppressed, in particular in the heavy quark limit. Theoretically the $b\bar{b}$ hybrid is most clean-cut but experimentally hard to produce.

On the lattice electromagnetic matrix elements can be calculated [23,29] but strong decays are very challenging. To this end McNeile *et al.* [28] predict that if the lightest bottomonium hybrid is indeed below the $B^{**}\overline{B}$ threshold the dominant decay channel should be deexcitation by emission of a scalar: $H_b \to \chi_b \pi\pi$, with a width of about 100 MeV. The same argument should also be valid in the charmonium case, where phase space would reduce the width even further, but not necessarily for light hybrids.

3 Conclusions: From fiction to fact

A combination of new theoretical methods and computing technology has allowed us to arrive at the boundary between qualitative test of principle and quantitative prediction in the complicated area of flavour singlet physics, strong decays and mixing. A few years ago at least three major technical challenges had to be overcome: light quarks, sea quarks and disconnected quark lines. We are about half way through by now. With enough effort devoted onto the topics covered in this article, quantitative predictions are possible within the time scale that is relevant for experiments like glueX at JLAB or PANDA at GSI.

Acknowledgements. I warmly thank Sigfrido Boffi, Claudio Ciofi degli Atti and Mauro Giannini for organizing this stimulating meeting. This work has been supported by PPARC grants PPA/A/S/2000/00271 and PPA/G/0/2002/0463.

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